An enhanced breaking waves model and morphodynamic coupling

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Introduction

To study morphodynamic evolution in the coastal area, it is important to have a reliable, accurate, and operational models to describe water wave propagation near the coast in the first place.

Objectives:

- Describe wave breaking phenomenon (Kazakova and Richard 2019)
- Preserve and improve the dispersive properties
- Have a hyperbolic reformulation (Richard 2021) to avoid dealing with mixed space-time high order derivative
- Couple with sediment transport equation to study the sandbar migration

Setup and assumption

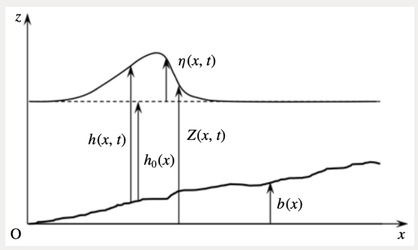


Figure 1. Setup of the model

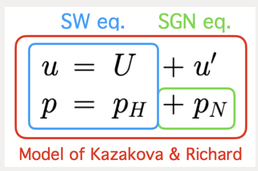


Figure 2. Assumptions in the variables

Assumptions:

- No surface tension and shear stress at the free surface
- The vertical variation of the filtered horizontal velocity is arbitrary (given by u')
- Hypothesis of weakly turbulent flow from Teshukov (2007) gives the order of u' as $O(\varepsilon)$ to include both shearing and dispersion
- The non-hydrostatic pressure is taken into account
- Sound velocity, a_c, is uniform and constant

Derivation

Asymptotic analysis with respect to the shallowness parameter

$$\varepsilon = \frac{h_0}{L} \ll 1,$$

where L, h_0 are the horizontal and vertical characteristic length, respectively.

Boundary conditions:

- No-penetration condition at the bottom
- Kinematic and dynamic condition at the free surface

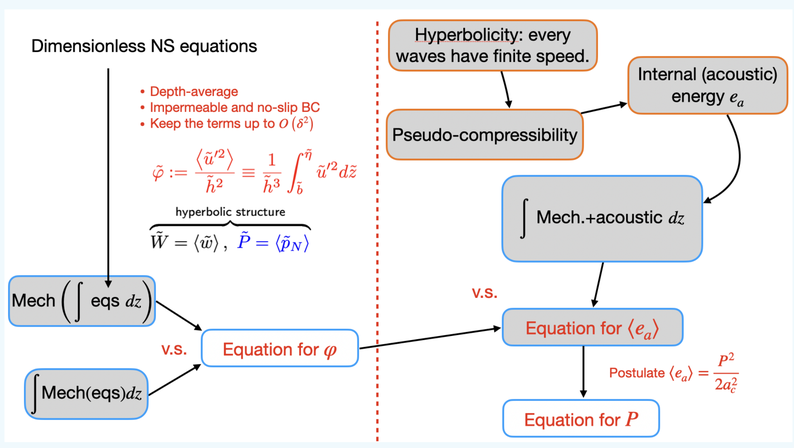


Figure 3. Sketch of the derivation

Final system of equation in mild slope

Under the mild slope assumption, we neglect the terms involving $\partial b/\partial x$ of order greater than $O(\varepsilon^3)$.

$$\begin{aligned}
&\frac{\partial h}{\partial t} + \frac{\partial hU}{\partial x} = 0 \\
&\frac{\partial hU}{\partial t} + \frac{\partial}{\partial x}\left(hU^2 + \frac{gh^2}{2} + h^3\varphi + hP\right) = \frac{\partial}{\partial x}\left(2\nu_T h\frac{\partial U}{\partial x}\right) - gh\frac{\partial b}{\partial x} \\
&\frac{\partial hW}{\partial t} + \frac{\partial hUW}{\partial x} = \frac{3}{2}P + 3\nu_T\frac{\partial U}{\partial x} \\
&\frac{\partial hP}{\partial t} + \frac{\partial hUP}{\partial x} = -a_c^2\left(h\frac{\partial U}{\partial x} + 2W\right) \\
&\frac{\partial h\varphi}{\partial t} + \frac{\partial hU\varphi}{\partial x} = -\frac{2}{h}\langle P^r\rangle + \frac{4\nu_T}{h}\left(\frac{\partial U}{\partial x}\right)^2 - \frac{8\nu_T W}{h^2}\frac{\partial U}{\partial x}
\end{aligned} \tag{1}$$

Blue: Shallow Water equation

Green: Incompressible hyperbolic SGN model developed by Richard (2021)

- The conservative part of the model is hyperbolic if $h > 0$
- Asymptotic dispersive properties to the SGN equation when $a_c \to \infty$
- A predictive law associated with breaking criterion is developed by Hung 2025
- 1D model with improved dispersive properties and 2D model have been derived

Hydrodynamic validation

The hydrodynamic model has been validated through many numerical test cases with various scenarios.

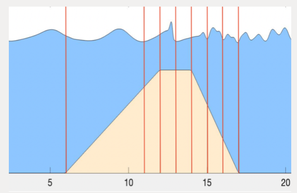


Figure 4. Experiment of Beji & Battjes consists of wave trains propagate over a submerged bar

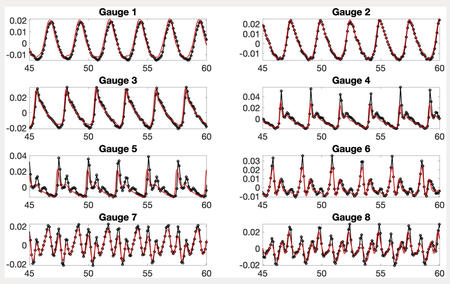


Figure 5. Time series comparison of the amplpitude at the gauge points. Numerical results in red solid line and experimental data in black markers.

Experimental Data on Sandbar Migration near coast

An important phenomenon affecting sandbar migration is the undertow. Undertow typically occurs beneath breaking waves and moves in an offshore direction.

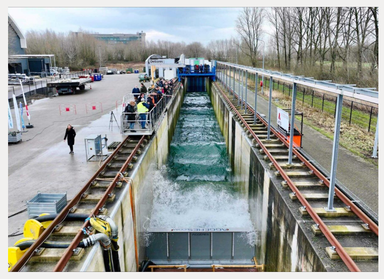


Figure 6. The European Large Installation Plan (LIP) experiments consist of complex wave conditions involving irregular wave trains over long durations.

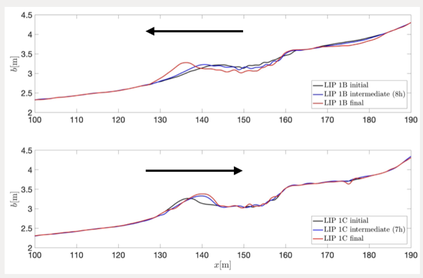


Figure 7. More energetic wave condition (top) creates greater undertow which migrates the sandbar in the offshore direction. (Adapt from Marchesiello *et. al.*, 2022)

Motivations: Does depth-averaged model capable to describe the undertow effect and predict the sandbar migration correctly?

Morphodynamic coupling and preliminary results

The concentration equation:

$$\frac{\partial hC}{\partial t} + \frac{\partial hUC}{\partial x} = E - D$$

Bed evolution equation (Exner-type):

$$(1 - n_p)\frac{\partial b}{\partial t} + \frac{\partial q_b}{\partial x} = D - E$$

where n_p is the bed porosity, q_b is the Meyer-Peter Müller bed-load flux.

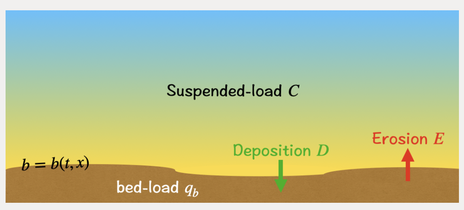


Figure 8. Sketch of sediment exchange near bed

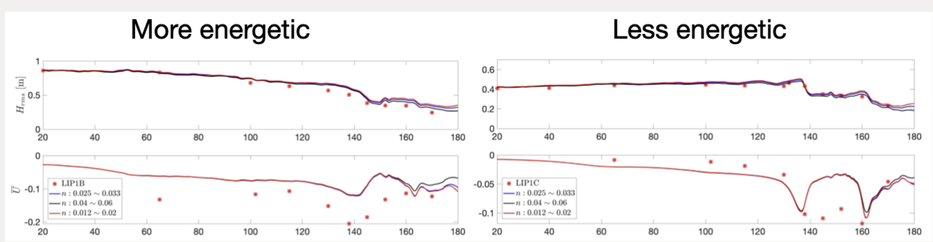


Figure 9. A comparison of the root-mean-square of the wave height H_{rms} (top) and the time-averaged horizontal velocity $\overline{U}$ (bottom) between the numerical results (solid line) and the experimental data (marker) for the two scenarios

Perspectives

- Bed evolution validation for the LIP experiment to demonstrate the model's capabilities in capturing sandbar migration.
- Systematic derivation of sediment transport equations from a multiphase flow model.
- Implementation of hydrodynamic model within TOLOSA using a new numerical scheme developed by Richard & Duran, 2024 for operational applications.

References

Hung, Y. C. (2025). "The enhanced breaking wave model and the morphodynamic coupling". PhD thesis. Université Savoie Mont Blanc.

Richard, G. L. (2021). "An extension of the Boussinesq-type models to weakly compressible flows". In: *European Journal of Mechanics - B/Fluids* 89, pp. 217–240. ISSN: 0997-7546. DOI: 10.1016/j.euromechflu.2021.05.011.

Kazakova, M. and G. L. Richard (2019). "A new model of shoaling and breaking waves: one-dimensional solitary wave on a mild sloping beach". In: *Journal of Fluid Mechanics* 862, pp. 552–591. DOI: 10.1017/jfm.2018.947.